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Space charge limited current flow between concentric spheres at potentials up to 15 MV

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Abstract. Poisson's equation in spherical symmetry is solved numerically with the assumptions of zero field and zero electron energy at the cathode surface. The computed current is expressed in terms of an analytic solution obtained for the extreme relativistic case. Internal and external anode configurations are treated, with ratio of electrode radii extending to 10 000 and potential difference up to 15 MV. Application of this analysis to the achievement of controlled fusion through pellet compression by charged particles is considered.

1. Introduction

The problem of current flow between concentric spheres has received little attention since the original non-relativistic treatments by Langmuir and Blodgett (1924), Langmuir and Compton (1931). Wheeler (1975) extended this treatment to include all values of field at the cathode surface, i.e. fields ranging from zero up to the space charge free value. Acton (1957) developed an approximate analytic expression for the space charge limited current, valid for potentials up to about 1 MV. The potential used in the calculations presented here extends up to 15 MV, which is the maximum potential currently employed in relativistic electron beam generators. It is assumed that the electrons are emitted from the cathode with zero energy and the mathematical technique employed is the same as that used by Wheeler (1977) for the corresponding cylindrical case. Perhaps the most important application of these calculations is to the problem of achieving a controlled thermonuclear reaction through fuel pellet compression using intense beams of relativistic electrons. Yonas *et al* (1974) envisaged the compression and heating of a small sphere of solid deuterium–tritium mixture by symmetric radiation of its surface with relativistic electrons. Clauser (1975) showed that the electron current required to achieve fusion is much reduced if the fuel is in a gaseous state and contained in a spherical shell of heavy metal, such as gold. The shell is sufficiently thick to fully stop the incident electrons and therefore it acquires a high collapse velocity which heats the contained fuel by adiabatic compression. Typical design parameters for break-even of energy are a pellet radius between and 1 and 2 mm, a gold shell thickness of 0·2 mm, an electron energy between 1 and 3 MeV, and an electron current between 100 MA and 1000 MA. For efficient transfer of energy the incident beam of electrons must be spherically symmetric to better than ±5% and the achievement of this degree of symmetry presents a practical problem. One line of approach is to make use of the bending of the electron trajectories caused by the self-magnetic field of the current. For example Chang *et al* (1975) experimented with a small anode sphere mounted by a stalk on one of a pair of large parallel plate electrodes. Although the measured anode current density was surprisingly symmetric, it still fell short of the theoretical requirement. Consequently for fusion purposes it may well prove necessary to use a near spherical arrangement of cathodes about a central anode fuel pellet. In such geometries the calculations presented here enable the cathode dimensions to be estimated for prescribed values of pellet radius, incident electron energy and electron current.

2. Mathematical formulation

Consider a concentric spherical geometry comprising an anode sphere, of radius r_1, maintained at a potential V_1 with respect to the cathode sphere, radius r_0. In the steady state the potential and space charge density ρ in the inter-electrode region are related through Poisson's equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}V}{\mathrm{d}r}\right) = -4\pi\rho. \tag{1}$$

If the electrons are emitted from the cathode with zero energy then ρ can be related to the total current I and also to the electron velocity v which, in turn, can be related to the local potential V

$$I = -4\pi r^2\rho v, \qquad mc^2(1-v^2/c^2)^{-1/2} = mc^2+eV. \tag{2}$$

These equations assume that the current I is sufficiently low for self-magnetic fields not to influence the electron motion. It is convenient to express radii and potentials in the following non-dimensional form

$$x = \ln(r/r_0), \qquad y = eV/mc^2. \tag{3}$$

Equations (1), (2) and (3) combine to give

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}B(1+y)(y^2+2y)^{-1/2}, \tag{4}$$

where $B = 2eI/mc^3$. In order to evaluate the space charge limited current this equation must be solved subject to zero field at the cathode surface, i.e. to $(\mathrm{d}y/\mathrm{d}x)_0 = 0$, assuming that the emission process causes no limitation. An analytic solution is possible in the following two simple cases. Firstly if $x \ll 1$ and $y \ll 1$, i.e. $|r-r_0| \ll r_0$ and $eV \ll mc^2$, then the right-hand side of equation (4) has a simple $y^{-1/2}$ dependence and double integration gives

$$x = 2^{7/4}3^{-1}B^{-1/2}y^{3/4}, \qquad x \ll 1, y \ll 1. \tag{5}$$

This is simply the Child–Langmuir relation for the current density, $I/4\pi r_0^2$, of non-relativistic electrons flowing between infinite plane electrodes separated by a distance $|r-r_0|$. Secondly if $y \gg 1$ for all x then the right-hand side of equation (4) is independent of y and double integration gives

$$B = 2y(\mathrm{e}^{-x}+x-1)^{-1}, \qquad y \gg 1. \tag{6}$$

This equation assumes that the electrons move with the velocity of light over the entire inter-electrode region, including the cathode surface. In terms of anode radius and anode potential this extreme relativistic approximation yields a current

$$I_r = cV_1[\ln(r_1/r_0) + (r_0/r_1) - 1]^{-1}. \quad (7)$$

Equation (4) can be expressed in a form suitable for general numerical solution by multiplying throughout by $2\,\mathrm{e}^{2x}\,\mathrm{d}y/\mathrm{d}x$ and integrating twice with respect to y:

$$x_1 = B^{-1/2} \int_0^{y_1} \mathrm{e}^x \left(\int_0^y \mathrm{e}^{2x}(1+y)(y^2+2y)^{-1/2}\,\mathrm{d}y \right)^{-1/2} \mathrm{d}y. \quad (8)$$

This equation is integrated numerically, from the cathode towards the anode, using the non-relativistic planar approximation of equation (5) as a first approximation for x in the integrand. The improved value of x so obtained is then used as a second approximation, and so on.

3. Results and discussion

For a prescribed value of B, i.e. specified total current I, the numerical integration yields sets of coordinates (x_1, y_1), i.e. coordinates $(r_1/r_0, V_1)$. For both external and internal anode, i.e. positive and negative x, solution for $10^{-6} \leqslant B \leqslant 10^2$ provides sufficient coordinates to cover a range of up to 10 000 in ratio of electrode radii. The most concise way of presenting these results is in the form of a numerical correction factor to be applied to the extreme relativistic approximation of equation (7). The parameter presented is the ratio I_r/I which always exceeds unity since the approximation I_r

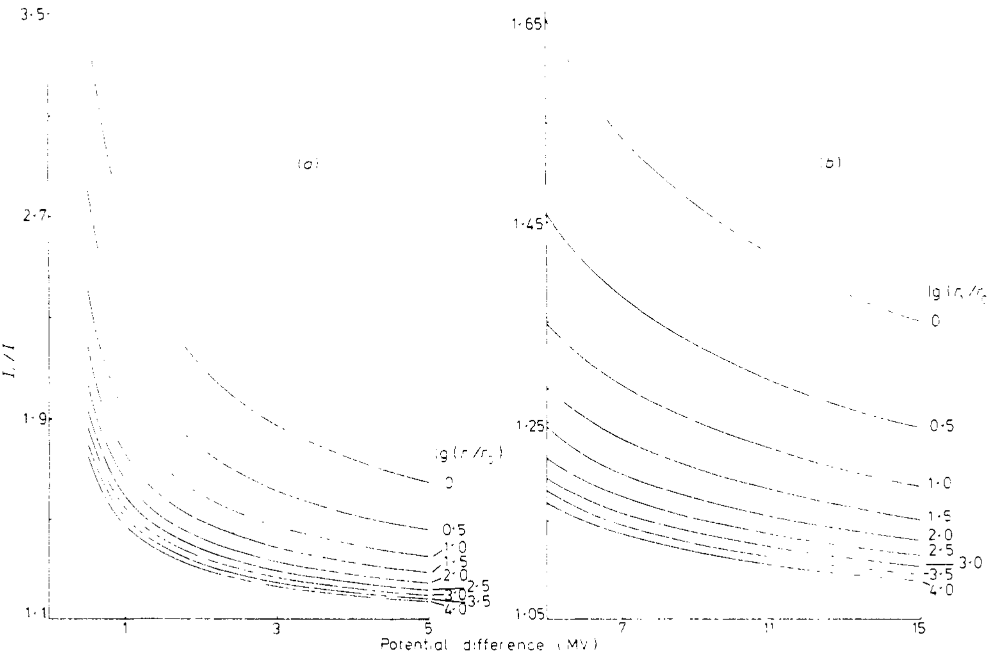

Figure 1. External anode. Calculated current I in terms of the approximation I_r of equation (7) as a function of voltage and electrode radii. (*a*) 0·5 MV to 5 MV; (*b*) 5 MV to 15 MV.

assumes an electron velocity c at the cathode whereas the current I is calculated for zero emission energy. Figures 1(a) and 1(b) show the results for an external anode with potential differences ranging from 0·5 MV to 15·0 MV. The ratio of radii is presented in multiples of $\sqrt{(10)}$, which is convenient for interpolation to any value in the range $1 \leqslant r_1/r_0 \leqslant 10\,000$. For the case of the internal anode the range of I_r/I embraced by the calculations is so great that it is necessary to present the results in the three figures, 2(a), 2(b) and 2(c). In both configurations the current I naturally approaches the extreme relativistic value I_r as the potential difference increases. However the approach is most rapid when the cathode curvature is greatest ($r_1/r_0 \rightarrow \infty$ for $r_1 > r_0$ or $r_0/r_1 \rightarrow 1$ for $r_1 < r_0$) since this is the condition that maximises the acceleration of the electrons at the beginning of their traverse. At the lowest potential presented, 0·5 MV, the non-relativistic calculations of Langmuir and Blodgett (1924) can considerably overestimate the current. This discrepancy amounts to 10% for comparable radii ($r_1/r_0 \rightarrow 1$), decreases for the internal anode as r_0/r_1 increases and increases for the external anode

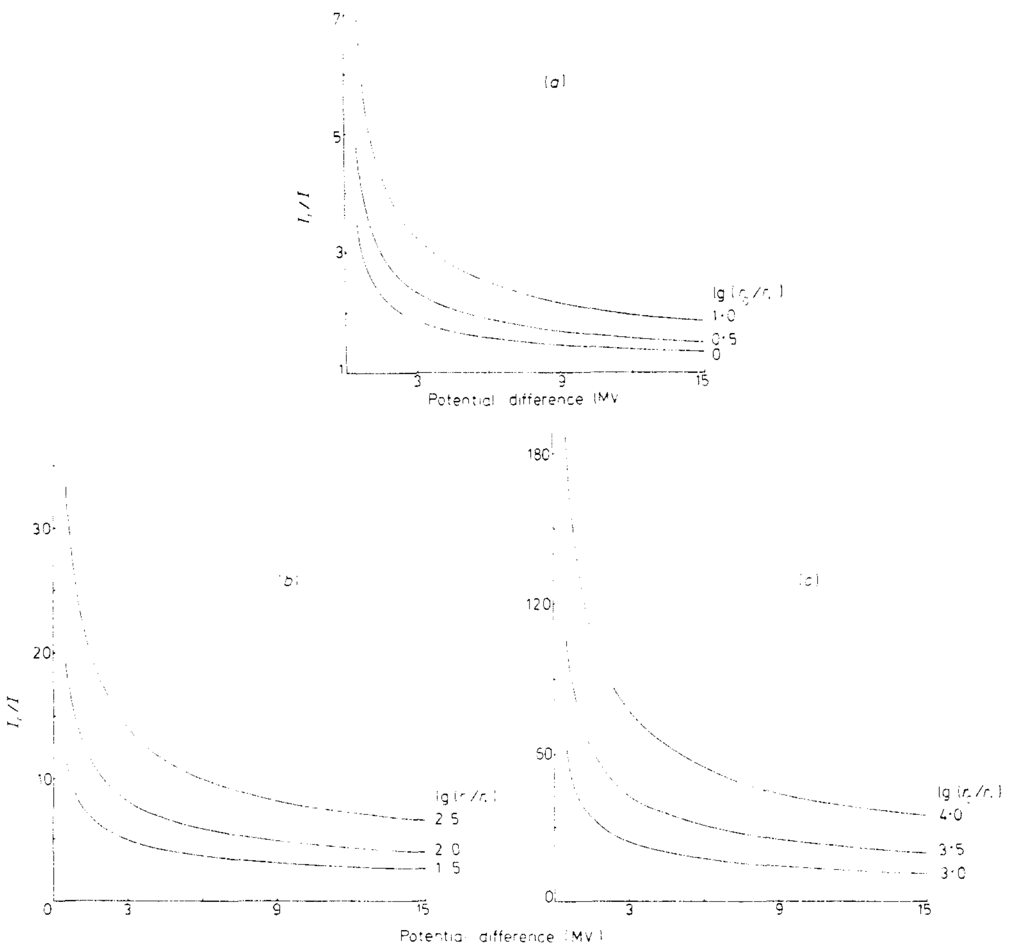

Figure 2. Internal anode. Calculated current I in terms of the approximation I_r of equation (7) as a function of voltage, 0·5 MV to 15 MV, and electrode radii. (a) $1 < r_0/r_1 < 10$; (b) $30 < r_0/r_1 < 300$; (c) $1000 < r_0/r_1 < 10\,000$.

as r_1/r_0 increases. In this latter configuration the discrepancy amounts to 30% at $r_1/r_0 = 10^4$.

Application of these calculations to the problem of anode pellet compression at 3 MV indicates that a current of 1 MA requires $r_0/r_1 = 1{\cdot}36$, 10 MA requires $r_0/r_1 = 1{\cdot}11$ and 100 MA requires $r_0/r_1 = 1{\cdot}033$. Such a small ratio of radii is completely unacceptable for fusion purposes since the cathode must be far removed from the anode pellet if it is not to be damaged by the fusion products. In any case if $r_1 = 2$ mm then the cathode radius r_0 must be at least an order of magnitude greater than this if flashover at 3 MV is to be avoided. Consequently the simple concept of a reactor composed of two concentric spherical electrodes in vacuum must be abandoned for fusion purposes. Two alternative schemes have been proposed that enable the cathode to be much further removed from the fuel pellet. The first approach involves surrounding the anode pellet by a spherical cloud of plasma that extends to many pellet radii. If the plasma is sufficiently ionised then the potential drop across it is very small and its outer surface is effectively at the anode potential. In several laboratories attempts are being made to project an externally produced plasma into the anode region of vacuum diodes. Alternatively the plasma might be produced internally by laser ablation of the pellet surface or even by ablation caused by the electron beam itself. The second approach envisages the pellet as a central target surrounded by a concentric spherical electron gun. In this arrangement the anode is a metal foil, transparent to relativistic electrons, situated close to and concentric with the surrounding cathode. It is hoped to prevent spreading of the electron beam between the anode and pellet by applying a suitable magnetic field or by introducing a low density gas. For both of these approaches, i.e. plasma anode or foil anode, the present calculations enable the cathode radius r_0 to be related to the electrode separation $(r_0 - r_1)$. At 3 MV a current of 1 MA requires $r_0 = 3{\cdot}8\,(r_0 - r_1)$, 10 MA requires $r_0 = 10(r_0 - r_1)$ and 100 MA requires $r_0 = 31(r_0 - r_1)$. It must be emphasised that this analysis does not include the effect of magnetic fields caused by the current flow. There is likely to be a very large, asymmetric, perturbing magnetic field associated with the return current lead to the pellet anode.

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